STC-MIMO Block Spread OFDM in Frequency Selective Rayleigh Fading Channels

Le Chung Tran, Xiaojing Huang, Eryk Dutkiewicz and Joe Chicharo

Wireless Technologies Laboratory (WTL)
Telecommunications & Information Technology Research Institute
Faculty of Informatics, University of Wollongong
Wollongong, NSW 2522, Australia
Email: {lct71,huang,eryk,chicharo}@uow.edu.au

Abstract—In this paper, we expand the idea of spreading the transmitted symbols in OFDM systems by unitary spreading matrices based on the rotated Hadamard or rotated Discrete Fourier Transform (DFT) matrices proposed in the literature to apply to Space-Time Coded Multiple-Input Multiple-Output OFDM (STC-MIMO-OFDM) systems. We refer the resulting systems to as STC-MIMO Block Spread OFDM (STC-MIMO-BOFDM) systems. In the proposed systems, a multi-dimensional diversity, including time, frequency, space and modulation diversities, can be used, resulting in better bit error performance in frequency selective Rayleigh fading channels compared to the conventional OFDM systems with or without STCs. Simulations carried out with the Alamouti code confirm the advantage of the proposed STC-MIMO-BOFDM systems.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) has been intensively considered in the literature for transmitting signals over frequency selective fading channels. The main advantage of the OFDM technique compared to a single carrier modulation is that it facilitates the use of high data rates with a relatively low complexity receiver, which requires only a Fast Fourier Transform (FFT) processor followed by single tap equalizers across all subcarriers. OFDM systems possess a diversity gain over single carrier systems since the orthogonality of the subcarriers prohibits multipaths from being combined across the channel at the symbol level.

The use of spreading matrices to mix the transmitted symbols linearly across the subchannels has been considered for single antenna wireless communications systems in the literature, such as in [1], [2]. The main advantage of using spreading matrices is that it allows us to achieve a diversity over the frequency selective fading channels. The use of unitary spreading matrices based on the rotated Hadamard or rotated Discrete Fourier Transform (DFT) matrices to improve the performance of single antenna OFDM systems was firstly introduced by Bury et.al. [2] and further studied by McCloud [3].

It is well known that the use of multiple transmit and/or receive antennas can significantly improve the capacity of wireless communications systems. Such systems are called Multiple Input Multiple Output (MIMO) systems. Space-Time Block Codes (STBCs) [4], [5], [6], Space-Time Trellis Codes (STTCs) [7], Bell Lab Layered Space-Time (BLAST) [8] are among various types of STCs. The combination of MIMO systems using STCs with the OFDM technique has received a fair amount of attention in the literature, such as in [9], [10], [11], [12], [13]. However, the combination of MIMO systems using STCs with the block spread OFDM technique has not been considered yet. This combination is expected to improve further the performance of the whole system at the cost of slightly more complicated transmitter and receiver structures.

In this paper, we expand the idea of Block Spread OFDM (BOFDM) mentioned in [2], [3] to apply to the MIMO-OFDM systems using STCs. We call the resultant technique STC-MIMO-BOFDM to distinguish it from the normal MIMO-OFDM without block spreading. It is noted that the term “spreading” is inherited from [2] to express the modulation alphabet expansion, rather than the bandwidth expansion as the normal meaning of this term.

The novel contributions of this paper include 1) the application of the block spreading technique to the conventional STC-MIMO-OFDM systems to improve further the error performance of the STC-MIMO-OFDM systems; 2) the derivation of the more detailed Simple Maximum Likelihood (SML) decoding method [14] for the proposed STC-MIMO-BOFDM systems using the Alamouti code [4]; and 3) the error performance comparison between the proposed STC-MIMO-BOFDM systems and the conventional OFDM systems (without STCs or block spreading), BOFDM systems (without STCs) [3] and STC-MIMO-OFDM systems (without block spreading) [14], [15].

The paper is organized as follows. In Section II, we derive the baseband model of an STC-MIMO-BOFDM system. Section III derives a detailed decoding algorithm, namely Simple Maximum Likelihood (SML), which was originally derived in [14], for a particular example of the Alamouti code. Simulation results are given in Section IV, and the paper is concluded by Section V.
II. STC-MIMO BLOCK SPREAD OFDM SYSTEM MODEL

The model of the baseband STC-MIMO-BOFDM systems is given in Fig. 1. We consider here an STC-MIMO-BOFDM system with \( N \) OFDM subcarriers, \( M_t \) transmit (Tx) antennas and \( M_r \) receive (Rx) antennas.

At each time \( t \), a block of information bits is encoded to generate a space-time code word consisting of \( T \times M_t \) modulated symbols as follows:

\[
S_t = \begin{bmatrix}
  s_{1,1}^t & \cdots & s_{1,M_t}^t \\
  \vdots & \ddots & \vdots \\
  s_{T,1}^t & \cdots & s_{T,M_t}^t
\end{bmatrix},
\]

where the symbols \( s_{k,i}^t \) are drawn from a signal constellation \( \mathcal{S} \). For simplicity, we select \( T = N \), that is, each modulated symbol modulates an OFDM subcarrier.

In contrast with the conventional STC-MIMO-OFDM systems, in our proposed STC-MIMO-BOFDM systems, each column of the code word \( S_t \) is divided into small blocks, each containing \( M \) symbols. Each block is then multiplied with a unitary spreading matrix \( U_M \). The data streams are then interleaved and OFDM modulated.

The idea of spreading the transmitted symbols with the unitary spreading matrices based on the rotated Hadamard or DFT matrices was firstly proposed in [2], and further examined for the BOFDM systems (single antenna systems without using STCs) in [3], and is expanded for the STC-MIMO-BOFDM system in this paper. The main idea of block spreading in the STC-MIMO-BOFDM systems is to split the full set of the subcarriers into smaller blocks and spread the transmitted symbols across these blocks via unitary spreading matrices in order to gain multipath diversity across each block at the receiver. In addition, the spreading transform using the rotated Hadamard or DFT matrices can be considered as a code in Euclidean space [2]. Block spreading using the rotated Hadamard or DFT matrices tends to expand the signal modulation alphabet and increase the Euclidean distance between sequences after being spread, compared to the normal Hadamard or DFT matrices [2]. In other words, besides having a multipath diversity, block spreading using the rotated Hadamard or DFT matrices also possesses a certain modulation diversity.

Consequently, a combined diversity, including time (STCs and interleaving), frequency (subcarriers), space (STCs) and modulation diversities (block spreading), can be used in the STC-MIMO-BOFDM systems, resulting in better bit error performance in frequency selective Rayleigh fading channels compared to the conventional OFDM systems with or without STCs.

The \( M \times M \)-size unitary spreading matrices \( U_M \) used in this paper can be either the rotated Hadamard matrix or the rotated DFT matrix which has been mentioned in [2]

\[
U_M = \mathbf{Z} \ diag\{\alpha_m\} \quad 0 \leq m \leq M - 1,
\]

where \( \mathbf{Z} \) denotes the diagonal matrix and \( \alpha_m = e^{j\theta_m} \), for \( 0 \leq m \leq M - 1 \), are selected to achieve diversity advantage, where \( \theta_m = \pi m / C \). Here \( C \) is chosen so that \( 2\pi / C \) is the smallest angle which rotates the signal constellation back to itself. For instance, we have \( C = 2^b \) for the signal constellation \( 2^b\)-PSK (\( b \) is an integer), while for QAM constellations such as 16-QAM or 64-QAM, we have \( C = 4 \). \( 
\mathbf{Z} \) is either the Hadamard matrix of order \( M \) or a DFT matrix of order \( M \) which is defined by its elements as [2]

\[
Z_{i,k} = \frac{1}{\sqrt{M}} \exp\left(-j2\pi \frac{(i-1)(k-1)}{M}\right).
\]

We represent the code word after being interleaved and OFDM modulated as

\[
X^t = \begin{bmatrix}
  x_{1,1}^t & \cdots & x_{1,M_t}^t \\
  \vdots & \ddots & \vdots \\
  x_{N,1}^t & \cdots & x_{N,M_t}^t
\end{bmatrix},
\]

where each symbol is formed by modulating a symbol in (1) with a certain subcarrier. The modulation is carried out by IFFT processors.

To avoid the inter symbol interference (ISI) due to the delay spread of the channel, a cyclic prefix (CP) is appended to each OFDM frame during a guard time interval (GI). CP is a copy
of the last \( L \) samples of each OFDM frame and is appended to the beginning of that frame, making the overall length of an OFDM frame equal to \((N + L)\). If we assume that the channel comprises \( L_P \) paths, then \( L \) should satisfy \( L \geq L_P \). After the CP is appended, \( \mathbf{X}' \) has a size of \((N + L) \times M_t\), which is transmitted via \( M_t \) Tx antennas and each column is the data sequence of \((N + L)\) symbols transmitted simultaneously in an OFDM frame.

Although the time domain model is conceptually straightforward, it is much more insightful to consider the system in the frequency domain. We assume that the channel is frequency selective and time-invariant (slow fading) during the transmission of one OFDM symbol. The frequency-domain model of the STC-MIMO-BOFDM system after removing the CP at time \( t \) is

\[
\mathbf{Y}_{k,j}^t = \sum_{i=1}^{M_t} H_{j,i}^{t,k} x_{k,i} + W_{k,j}^t, \tag{5}
\]

where \( H_{j,i}^{t,k} \) is the channel frequency response for the path from the \( i \)-th Tx antenna \((i = 1, \ldots, M_t)\) to the \( j \)-th Rx antenna \((j = 1, \ldots, M_r)\) on the \( k \)-th OFDM subcarrier \((k = 1, \ldots, N)\) and \( W_{k,j}^t \) is a noise sample at the output of OFDM demodulators (FFT). \( W_{k,j}^t \) is the \( N \)-point discrete Fourier transform of the corresponding discrete time noise sample, and can be modeled as a complex Gaussian random variable with zero mean and power \( N_0/2 \) per dimension. It is known that \( H_{j,i}^{t,k} \) can be calculated as

\[
H_{j,i}^{t,k} = \mathbf{h}_{j,i}^t \mathbf{w}_k, \tag{6}
\]

with

\[
\mathbf{h}_{j,i}^t = [h_{j,i}^{t,1}, h_{j,i}^{t,2}, \ldots, h_{j,i}^{t,L_P}],
\mathbf{w}_k = [e^{-2\pi k/N}, e^{-2\pi 2k/N}, \ldots, e^{-2\pi L_P k/N}]^T, \tag{7}
\]

where \( h_{j,i}^{t,l} \) is the impulse response of the channel between the \( i \)-th Tx antenna and the \( j \)-th Rx antenna in the \( l \)-th multipath \((l = 1, \ldots, L_P)\) at time \( t \) and \((.)^H\) denotes the transposition operation. In this paper, the channel coefficients \( H_{j,i}^{t,k} \) are assumed to be known at the receiver.

### III. SML Decoding Algorithm

In this section, we illustrate an example of the two Tx antenna (i.e. \( M_t = 2 \)) and one Rx antenna (\( M_r = 1 \)) STC-MIMO-BOFDM system using the Alamouti code [4] given below

\[
\begin{bmatrix}
    x_1 & -x_2^* \\
    x_2 & x_1^*
\end{bmatrix}, \tag{8}
\]

where \( * \) denotes the complex conjugate. In Eq. (8), the two columns are transmitted via the two Tx antennas while the two symbols in each column are modulated by the two consecutive \( k \)-th and \( (k + 1) \)-th subcarriers. At the receiver, the CP is removed. The received signals are then deinterleaved and despread by multiplying the resultant signals with the inverse unitary spreading matrix \( \mathbf{U}_M^{-1} \).

For simplicity, we still use similar notations as in Eq. (5) for the received signals after being deinterleaved and despread. Let us consider an STC-MIMO-BOFDM frame consisting of \( N \) symbols \((N \) is normally chosen to be a power of 2 to facilitate the IFFT and FFT). As a certain OFDM frame is being considered and the system comprises one Rx antenna, for the ease of exposition, we omit the superscript \( t \) and subscript \( j \) in (5). The transmitted symbols in the OFDM frame via two Tx antennas are expressed as

\[
\begin{bmatrix}
    x_1 & -x_2^* & \ldots & -x_{N-1}^* & x_N\\
    x_2 & x_1^* & \ldots & x_{N-1} & x_{N-1}^*
\end{bmatrix}^T, \tag{9}
\]

where symbols in each column are simultaneously transmitted via one Tx antenna in one OFDM frame. If we denote

\[
\begin{bmatrix}
    H_1^t & H_2^t & 0 & 0 & 0 \\
    H_2^t & -H_1^t & 0 & 0 & 0 \\
    0 & 0 & 0 & H_{N-1}^t & H_{N-1}^t \\
    0 & 0 & 0 & -H_{N-1}^t & -H_{N-1}^t
\end{bmatrix},
\mathbf{H} =
\begin{bmatrix}
    W_1 & W_2 & \ldots & W_{N-1} & W_N
\end{bmatrix}^T,
\mathbf{X} = [x_1, x_2, \ldots, x_N]^T,
\mathbf{Y} = [Y_1, Y_2^*, \ldots, Y_{N-1}, Y_N^*]^T, \tag{10}
\]

then Eq. (5) can be equivalently expressed as follows:

\[
\mathbf{Y} = \mathbf{HX} + \mathbf{W}. \tag{11}
\]

In (10), the superscript of \( H^t \) indicates the subcarrier index while the subscript indicates the Tx antenna index. By multiplying both sides of Eq. (11) with a matrix \( \mathbf{C}_N = \mathbf{A}_N \mathbf{H}^H \), we have

\[
\begin{align*}
\mathbf{Y}_N &= \mathbf{C}_N \mathbf{Y} = \mathbf{A}_N \mathbf{H}^H \mathbf{X} + \mathbf{A}_N \mathbf{H}^H \mathbf{W}, \\
\mathbf{Y}_N &= \mathbf{H}_N \mathbf{X} + \mathbf{W}_N, \tag{12}
\end{align*}
\]

where \((.)^H\) denotes the Hermitian transpose operation; \( \mathbf{W}_N \triangleq \mathbf{A}_N \mathbf{H}^H \mathbf{W} \); the matrix \( \mathbf{A}_N \) is selected so that the diagonal elements of \( \mathbf{E} (\mathbf{W}_N \mathbf{W}_N^H) \) are \( N_0 \) and given by

\[
\mathbf{A}_N = \text{diag}(\alpha_1^{-1/2}, \alpha_2^{-1/2}, \ldots, \alpha_N^{-1/2}), \tag{13}
\]

and

\[
\mathbf{H}_N = \mathbf{A}_N \mathbf{H}^H \mathbf{H} =
\begin{bmatrix}
    \alpha_1^{1/2} & \beta_1 \alpha_1^{-1/2} & 0 & \ldots & 0 \\
    \beta_1^* \alpha_1^{1/2} & \alpha_1^{1/2} & 0 & \ldots & 0 \\
    0 & 0 & \alpha_2^{1/2} & \ldots & \alpha_2^{-1/2} \\
    \ldots & \ldots & 0 & \ldots & \ldots \\
    0 & 0 & \ldots & \alpha_{N-1}^{-1/2} & \beta_{N-1,N} \alpha_{N-1}^{1/2} \\
    0 & 0 & \ldots & \ldots & \alpha_{N-1}^{-1/2} \\
\end{bmatrix}, \tag{14}
\]

where

\[
\alpha_k = |H_k^t|^2 + |H_{k+1}^t|^2, \\
\alpha_{k+1} = |H_k^{t+1}|^2 + |H_{k}^{t+1}|^2, \\
\beta_{k,k+1} = H_k^{t+1*} H_k^{t+1} - H_k^{t+1*} H_k^{t+1}, \tag{15}
\]

and
for \( k = 1, 3, 5, \ldots, N - 1 \).

In this paper, we use the SML decoding method [14] for our proposed STC-MIMO-BOFDM systems in the considered scenario. The main reasons for this are: 1) the computational complexity for decoding signals is linearly proportional to the size of OFDM frames and is much simpler than other decoding methods such as the sphere decoding method [16], and 2) the computational complexity for decoding signals is linearly proportional to the number of Tx (or Rx) antennas, rather than roughly cubic as in the sphere decoding method, in the case of \( M_t = M_r \) [16]. Therefore, by using the SML decoding method, the computational effort is significantly reduced, facilitating the utilization of large numbers \( N \) and \( M \) as well as large size signal constellations \( \mathcal{S} \).

In the SML decoding method, the transmitted signals are decoded without considering the correlation of the noise \( \mathbf{W}_N \) and the crosstalk, that is, the off-diagonal elements of the matrix \( \mathbf{H}_N \) in (14). Therefore, the symbols transmitted in an OFDM frame can be decoded separately, rather than jointly, as follows:

\[
\hat{x}_k = \arg \min_{x \in \mathcal{S}} \{ |Y_k - \alpha_k^{1/2} x|^2 \}, \tag{16}
\]

for \( k = 1, \ldots, N \). In (16), \( Y_k \) denotes the \( k \)-th element of \( \mathbf{Y}_N \).

This decoding algorithm comes at the cost of inferior error performance compared to other complicated decoding methods, such as Joint Maximum Likelihood (JML), Zero Forcing (ZF), Decision Feedback (DF) [14]. Consequently, the simulation results derived in this paper just aim at giving the readers the lower bound of the performance of the considered system. In other words, the advantage of our proposed STC-MIMO-BOFDM system can be improved even further if we use more complicated decoding methods.

### IV. Simulation Results

In this section, we consider an STC-MIMO-BOFDM system using the Alamouti code with 2 Tx antennas and 1 Rx antenna. Several Monte-Carlo simulations are carried out based directly on the analysis mentioned previously in Section III. Parameters used for the simulations are given in Table I. We compare the bit error performance of our proposed STC-MIMO-BOFDM system with that of three other systems, namely conventional OFDM system (without STCs or block spreading), BOFDM system (without STCs) [3] and STC-MIMO-OFDM system (without block spreading) [14], [15]. We assume that the Tx antennas are uncorrelated, that is, those antennas are separated far enough from each other so that the fading processes affecting those antennas can be considered to be independent. The interleaver and deinterleaver are utilized in all simulations. The signal-to-noise ratio \( SNR \) here means the average \( SNR \) per symbol. The channel is considered to be quasi-static so that the channel coefficients are constant during each OFDM frame. To model the frequency selective multipath Rayleigh fading channel, the time delay \( \Delta \tau_l \) between two consecutive \( (l + 1) \)-th and \( l \)-th paths of the channel is assumed to be

\[
\text{TABLE I}
\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of subcarriers ( N )</td>
<td>64</td>
</tr>
<tr>
<td>Cyclic prefix ( L )</td>
<td>16</td>
</tr>
<tr>
<td>Bandwidth (Sampling frequency) ( F_s )</td>
<td>20 MHz</td>
</tr>
<tr>
<td>Number of multipaths ( L_P )</td>
<td>3, 4 or 9</td>
</tr>
<tr>
<td>Maximum Doppler frequency ( F_D )</td>
<td>50 or 100 Hz</td>
</tr>
<tr>
<td>Power delay profile (pdp)</td>
<td>Exponential decay</td>
</tr>
<tr>
<td>Size of spreading matrix ( M )</td>
<td>16 or 32</td>
</tr>
<tr>
<td>Spreading matrix ( \mathbf{U}_M )</td>
<td>Rotated Hadamard or Rotated DFT matrix</td>
</tr>
<tr>
<td>Signal constellation ( \mathcal{S} )</td>
<td>BPSK, QPSK, 4QAM or 8PSK</td>
</tr>
</tbody>
</table>

![Fig. 2. Performance of the Alamouti STC-MIMO-BOFDM system with \( L_P = 3, M = 16, F_D = 100 \) Hz, and a BPSK constellation.](image1)

![Fig. 3. Performance of the Alamouti STC-MIMO-BOFDM system with \( L_P = 3, M = 16, F_D = 100 \) Hz, and a QPSK (or 4QAM) constellation.](image2)
equal to the sampling period $T_s = 1/F_s$, i.e. $\Delta T_l = T_s$
for $l = 1, \ldots, L_P - 1$. We also assume that the channel
power delay profile (pdp) follows the exponential decay law $[1, e^{-1}, \ldots, e^{-\left(L_P-1\right)}].$

Fig. 2 shows the case where the size of spreading matrices $M = 16$, the number of multipaths $L_P = 3$, the maximum
Doppler frequency $F_D = 100$ Hz, and the BPSK constellation are simulated. We can see from the figure that the use of block
spreading provides the improvement of the bit error performance in both single antenna and multiple antenna systems (or
MIMO systems). The proposed STC-MIMO-BOFDM system provides the best performance among the considered systems.
In particular, it provides an 1.2 dB $SNR$ gain over the STC-MIMO-OFDM system at $BER = 10^{-5}$.

In Fig. 3, we consider the performance of the three systems STC-MIMO-BOFDM, STC-MIMO-OFDM and BOFDM in
the scenario where $M = 16$, $L_P = 3$, $F_D = 100$ Hz, and the QPSK (or 4QAM) constellation are simulated. An 1 dB $SNR$
gain can be achieved by the proposed STC-MIMO-BOFDM system at $BER = 10^{-5}$, compared to the conventional STC-
MIMO-OFDM system, while a 5dB gain can be achieved by the STC-MIMO-BOFDM system, compared to the conven-
tional BOFDM system.

The scenario where $M = 32$, $L_P = 4$, $F_D = 100$ Hz, and an 8PSK constellation are simulated is shown in Fig. 4. Similarly, the proposed STC-MIMO-BOFDM system provides the best bit error performance with a 1.5 dB $SNR$ gain
over the conventional STC-MIMO-OFDM system at $BER = 10^{-2}$ and with a 7 dB gain over the BOFDM system. All
aforementioned simulations are carried out with the rotated Hadamard matrices which are calculated following (2).

Finally, in Fig. 5, we simulate the three systems with $M = 16$, $L_P = 9$, $F_D = 50$ Hz, a QPSK (or 4QAM)
 constellation and the rotated DFT matrix whose elements are calculated following (3). Again, the proposed STC-MIMO-
BOFDM system provides the best bit error performance with a 2 dB $SNR$ gain over the conventional STC-MIMO-OFDM
system at $BER = 10^{-5}$.

Interestingly, we can see from Fig. 5 that the performance curve of the BOFDM systems using block spreading comes close to that of the MIMO-OFDM systems (without block
spreading) at high $SNRs$. Therefore, we can conclude that BOFDM systems may provide better bit error performance than MIMO-OFDM systems in some scenarios. This also
means that the former may possess higher diversity orders than the latter at high $SNRs$.

V. CONCLUSIONS

In this paper, we have expanded the idea of using block
spreading to improve the bit error performance of OFDM
systems over frequency selective multipath Rayleigh fading
channels to apply to STC-MIMO-OFDM systems, resulting
in so-called STC-MIMO-BOFDM systems. Simulations show
that the proposed STC-MIMO-BOFDM systems provide con-
siderable improvements in the bit error performance compared
to the conventional STC-MIMO-OFDM systems, and provide
much better performance, compared to the BOFDM and conventional OFDM systems. This advantage comes at the
cost of a slightly more complicated system structure.

REFERENCES

spread CDMA versus DS-CDMA for cellular downlink: a comparative
2004.
transforms for multicarrier spread spectrum transmission,” IEEE Trans.
matrices for use on multipath fading channels,” IEEE Trans. Commun.,


