Statistical Analysis of Interference in Asynchronous MC-CDMA Systems

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Abstract

Two major sources of interference affect asynchronous MC-CDMA systems, i.e. multiple access interference due to subcarriers with the same frequency (MAI) and multiple access interference due to subcarriers with different frequency (ICI). Both MAI and ICI are generally modeled as zero-mean Gaussian random variable and their power has been previously been derived in the case of uniformly distributed timing offsets. In this paper, we derive an expression of the conditional power of the MAI and ICI as a function of timing offset. The advantage is that the interference power can then be derived for various distributions of the timing offsets. We then apply the expression to calculating the MAI and ICI power for two different distributions of timing offsets, i.e. uniform distribution and Poisson distribution. Finally, we propose a statistical model for asynchronous MC-CDMA systems that will simplify the computer simulation process of these systems. It is based on modeling the asynchronous system with a synchronous system followed by additive noise representing the MAI and ICI. The model is validated by comparing the BER at the output of the asynchronous system and the model.

1. Introduction

MC-CDMA systems [1] have recently attracted a lot of research in the world of wireless telecommunications. This is mainly due to the fact that MC-CDMA systems have several advantages over current DS-CDMA systems, such as good spectrum properties and robustness to frequency selective fading. However, under asynchronous transmission, the performance of MC-CDMA systems is significantly affected by multiple access interference (MAI) [2].

Asynchronous transmission refers to the scenario whereby multiple user signals arrive at the receiver with different delays which is also referred as timing offsets [3]. Asynchronous transmission is at the source of two types of interference in MC-CDMA systems. MAI is the interference created by other users transmitting information over the same sub-carrier frequencies which is also common to DS-CDMA systems. ICI is more specific to MC-CDMA systems and represents the interference generated by transmission over sub-carriers with different frequency. The sum of the MAI and ICI is a good approximation of the total interference affecting asynchronous MC-CDMA systems. One aim of this paper is to estimate the total interference power for different statistics of timing offsets.

Previous studies devoted to analysing interference in asynchronous MC-CDMA systems generally assume the timing offset ‘τ’ to be uniformly distributed over one symbol duration $T_s$ [4]-[8]. In many applications, however, timing offsets are modelled as Poisson distributed [9]-[12]. In this paper, we derive an exact expression for the total interference power as a function of offset. The derived expression will allow the estimation of interference power for various distributions of timing offsets, including Poisson distribution. This result is then used to propose a computer simulation model for asynchronous MC-CDMA systems. Current models require heavy computational load. We propose to replace the asynchronous system with a synchronous system, which requires significantly less computing, followed by an additive noise component representing the MAI and ICI. The model is validated using computer simulations.

This paper is structured as follows. In section 2, we present the asynchronous MC-CDMA system considered in this paper. In section 3, we derive the total interference power affecting asynchronous MC-CDMA systems as a function of timing offsets. In Section 4, we calculate the interference power for two different distributions of the timing offsets, i.e. the uniform distribution and the Poisson distribution. The computer simulation model for asynchronous MC-CDMA systems is proposed in Section 5. Section 6 concludes this paper.

2. Asynchronous MC-CDMA Model

The asynchronous MC-CDMA model considered in this paper is shown in Figure 1. We consider $K$ users transmitting data assumed i.i.d and BPSK modulated, then spread in the frequency domain. We also assume the total
number of sub-carriers for each user is equal to the spreading factor.

The transmitted signal for the \( k \)th user is given by
\[
s_k(t) = \sum_{i=1}^{N} b_k(t) c_{k,i}(t) \cos(2\pi f_it) \tag{1}
\]
where \( N \) is the spreading factor; \( b_k(t) \) is the transmitted data bit; \( c_{k,i}(t) \) is the spreading code for the \( i \)th subcarrier in the \( k \)th user; \( f_i \) is the carrier frequency for the \( i \)th subcarrier given by \( f_i = f_c + \frac{i}{T_s} \) when \( i = 1, 2,...,N \) and \( f_c \) is the fundamental transmission frequency.

\[ r_k(t) = \sum_{i=1}^{N} \alpha_{k,i} b_k(t - \tau_k) c_{k,i}(t - \tau_k) \cdot \cos(2\pi f_i(t - \tau_k) + \psi_{k,i}) \tag{3}\]

3. Interference analysis

3.1 Multiple access interference

In the rest of the paper, the reference user is referred to as user \( h \). The MAI introduced by the \( k \)th user over the \( i \)th subcarrier path over one symbol duration is given by
\[
I_{MAI,j,k} = \int_0^{T_s} \alpha_{k,j} b_k(t - \tau_k) \cdot c_{k,j}(t) \cos(2\pi f_i(t - \tau_k)) dt \tag{4}
\]
By calculating the above integral, we find
\[
I_{MAI,j,k} = \frac{1}{2} c_{k,j}(T_s) \cos \theta_{k,j} - \left[ \alpha_{k,j-1} b_{k,j-1} + \alpha_{k,j,0} b_{k,j,0} (T_s - \tau_k) \right] \tag{5}
\]
where \( \theta_{k,j} = 2\pi f_i \tau_k - \psi_{k,j} \) is a phase offset, \( \alpha_{k,j-1} \) and \( \alpha_{k,j,0} \) are the channel Rayleigh fading parameter affecting the previous and current data bit respectively.

The total MAI affecting the reference user is the sum of the MAI contributions from each user and each subcarrier:
\[
I_{MAI,jot} = \sum_{k=1}^{K} \sum_{i=1}^{N} I_{MAI,j,k} \tag{6}
\]

3.2 Inter-Channel Interference

The ICI introduced by the sub-carriers of the \( k \)th user into the \( i \)th subcarrier of the reference user \( h \) over one symbol duration is given by
\[
I_{ICI,k,i} = \sum_{j=1}^{N} I_{ICI,k,i,j} \tag{7}
\]
where $I_{ICI,i,j,k}$ represents the ICI introduced by the $j$th subcarrier of user $k$ into the $i$th subcarrier of the reference user. It is given by [2]

$$I_{ICI,i,j,k} = \int_0^{\tau_i} \alpha_{k,j} \beta_t \left( t - \tau_k \right)$$

$$+ c_{h,i}(t) c_{h,j}(t) \cos \left[ 2 \pi f t \right]$$

$$- \cos \left[ 2 \pi f (t - \tau_k) + \psi_{k,j} \right]$$

The above integral can be expressed as

$$I_{MAIDF,i,j,k} = \frac{1}{2} \left[ R_{i,j}(\tau_k) + \tilde{R}_{i,j}(\tau_k) \right]$$

with

$$R_{i,j}(\tau_k) = \int_0^{\tau_i} \alpha_{k,j} \beta_t \left( t - \tau_k \right)$$

$$\cdot \cos \left[ 2 \pi (f_i - f_j) t + \theta_{k,j} \right] dt$$

$$R_{i,j}(\tau_k) = \int_0^{\tau_i} \alpha_{k,j} \beta_t \left( t - \tau_k \right)$$

$$\cdot \cos \left[ 2 \pi (f_i - f_j) t + \theta_{k,j} \right] dt$$

After evaluation of the integral, the total ICI for the $i$th subcarrier due to user $k$ is obtained as

$$I_{ICI,i,j,k} = \sum_{j=1}^{N} \frac{T_c c_{k,j}(t)c_{h,i}(t)}{4\pi(i-j)}$$

$$\cdot \left[ \alpha_{k,j} \beta_t - \alpha_{k,j} \beta_t \right]$$

$$\cdot \left[ \sin \left( \frac{2\pi (i-j)}{T_i} \tau_k + \theta_{k,j} \right) \right]$$

The total ICI affecting the reference user is the sum of the ICI contributions from each user, i.e.

$$I_{ICI,ref} = \sum_{k=1}^{K} \sum_{i=1}^{N} \sum_{j=1}^{N} h_{j,k}$$

(13)

### 4. 2nd Moment of Interference

The total interference of asynchronous MC-CDMA system is generally assumed Gaussian distributed with zero mean [2]. In order to obtain the expression of second moment of MAI in terms of the timing offset, $\tau_k$ for user $k$, we take the expectation with respect to $b(t)$, $\alpha$ and $\theta$ conditionally to $\tau_k$.

$$I_{MAI,ref}(\tau_k) = E_b[|\alpha|^2] \sum_{k=1}^{K} N^2 I_{MAI,k}^2 |\tau_k|$$

$$= E_b[|\alpha|^2] \sum_{k=1}^{K} N^2 I_{MAI,k}^2 |\tau_k|$$

(14)

$$= \sum_{k=1}^{K} E_b[|\alpha|^2] I_{MAI,k}^2 |\tau_k|$$

(15)

$$= \frac{\beta^2}{4} \sum_{k=1}^{K} \sum_{i=1}^{N} \left[ \tau_i^2 + (\tau_i^2 - \tau_i)^2 \right]$$

Hence the total interference power in terms of $\tau$ is given as

$$I_{ICI,ref}(\tau) = I_{MAI,ref}(\tau) + I_{ICI,ref}(\tau)$$

(16)

#### 4.1 Case 1: $\tau_k$ is uniformly distributed over $[0, T_i]$.

By taking the expectation of (12) and (13) further with respect to $h(t)$, $\alpha$ and $\theta$ conditionally to $\tau_k$. Similarly, the ICI power for reference user $h$ can be written as

$$I_{ICI,ref}(\tau_k) = \frac{T_c^2 \beta_t^2}{4\pi^2} \sum_{k=1}^{K} \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{1}{(i-j)^2}$$

$$\cdot \left[ 1 - \cos \left( \frac{2\pi(i-j)}{T_i} \tau_k \right) \right]$$

(15)

The total interference power in terms of $\tau$ is given as

$$I_{ICI,ref}(\tau) = I_{MAI,ref}(\tau) + I_{ICI,ref}(\tau)$$

(16)

#### 4.2 Case 2: $\tau_k$ is Poisson distributed.
The conditional probability density function (pdf) for Poisson distribution is given as 
\[ f(x / \lambda) = e^{-\lambda} \frac{\lambda^x}{x!} \]  
(20)

Where \( \lambda \) is the Poisson distribution parameter and is assumed to be the same for all channels. \( \hat{\lambda} \) also represents the mean and variance of the distribution and we restrict it, in this paper, to the interval \([0 T_s]\). Hence, the timing offsets lie, on average, within \([0 T_s]\), but can, in theory, take any value between 0 to \( \pm \infty \). As a result, interference does not only affect the current and previous information bits as in the uniform distributed case, but can also affect all the preceding bits. Our simulation results show, however, that as long as \( \hat{\lambda} \) is within the range of \([0 T_s]\), we can achieve a good approximation for the total interference power by only considering the current bit \( 'b_i' \), and the previous two bits \( 'b_{i-1}' \) and \( 'b_{i-2}' \). Further, the closer \( \lambda \) is to 0, the less significant the timing offsets outside \([0 2T_s]\) and the smaller the approximation error. These results are displayed in Table 1, in the next section. We now derive the interference power in the Poisson case, by restricting the interference to the two previous bits.

The interval \([0 T_s]\) is first sampled using \( M \) samples. The need for sampling arises from the need for using integer values in the Poisson distribution. The conditional probability density function (pdf) for Poisson distribution is given by

\[ P(x) = \frac{e^{-\lambda} \lambda^x}{x!} \]

For the case when \( \tau \) in the range of \((T_s, 2T_s)\), we again replace \( \tau_i \) in (22) with \((\tau_i - M)\).

By comparing (23) and (24), we find that these two formulas are effectively the same. Hence, we can combine (23) and (24), which gives,

\[ E[I_{2,\text{ICI, tot, } \tau_i = \{T_s, 2T_s\}}] = \sum_{\tau_i = \{T_s, 2T_s\}} E[I_{2,\text{ICI, tot, } \tau_i = \{0, T_s\}}] \]

(26)

We then obtain the 2nd order moment of the total interference by summing up equations (21), (22) and (25) which gives

\[ E[I_{2,\text{ICI, tot}}] = E[I_{2,\text{ICI, ICI, tot, } \tau_i = \{0, T_s\}}] + E[I_{2,\text{ICI, ICI, tot, } \tau_i = \{T_s, 2T_s\}}] \]

for the case when \( \tau \) is in the range of \([T_s, 2T_s]\), we can develop the expression of the 2nd moment ICI as follows

\[ E[I_{2,\text{ICI, ICI, tot, } \tau_i = \{0, T_s\}}] = \sum_{\tau_i = \{0, T_s\}} E[I_{2,\text{ICI, ICI, tot, } \tau_i = \{0, T_s\}}] \]

(22)

\[ = \frac{(K-1)T^2 \beta^2}{4\pi^2} \sum_{\tau_i = 0}^{T_s} \sum_{j=1}^{N} \sum_{k=1}^{M} \frac{1}{|1 - \cos(\frac{2\pi(j-2\tau_i)}{M})|} \frac{\lambda^2}{\tau_i^2} \]

(24)

For the case when \( \tau \) is in the range of \([T_s, 2T_s]\), we again replace \( \tau_i \) in (22) with \((\tau_i - M)\).

By comparing (23) and (24), we find that these two formulas are effectively the same. Hence, we can combine (23) and (24), which gives,

\[ E[I_{2,\text{ICI, ICI, tot, } \tau_i = \{T_s, 2T_s\}}] = \sum_{\tau_i = \{T_s, 2T_s\}} E[I_{2,\text{ICI, ICI, tot, } \tau_i = \{T_s, 2T_s\}}] \]

(25)

In the range \([0 T_s]\), we can simply replace \( \tau_i \) in (19) with \((\tau - M)\), which gives

\[ E[I_{2,\text{ICI, ICI, tot, } \tau_i = \{0, T_s\}}] = \sum_{\tau_i = \{0, T_s\}} E[I_{2,\text{ICI, ICI, tot, } \tau_i = \{0, T_s\}}] \]

(23)

\[ = \frac{(K-1)T^2 \beta^2}{4\pi^2} \sum_{\tau_i = 0}^{T_s} \sum_{j=1}^{N} \sum_{k=1}^{M} \frac{1}{|1 - \cos(\frac{2\pi(j-2\tau_i)}{M})|} \frac{\lambda^2}{\tau_i^2} \]

(21)

For the case when \( \tau \) is in the range of \((T_s, 2T_s)\), we can simply replace \( \tau_i \) in (19) with \((\tau - M)\), which gives

\[ E[I_{2,\text{ICI, ICI, tot, } \tau_i = \{T_s, 2T_s\}}] = \sum_{\tau_i = \{T_s, 2T_s\}} E[I_{2,\text{ICI, ICI, tot, } \tau_i = \{T_s, 2T_s\}}] \]

(24)

\[ = \frac{(K-1)T^2 \beta^2}{4\pi^2} \sum_{\tau_i = T_s}^{2T_s} \sum_{j=1}^{N} \sum_{k=1}^{M} \frac{1}{|1 - \cos(\frac{2\pi(j-2\tau_i)}{M})|} \frac{\lambda^2}{\tau_i^2} \]

(22)
\[ (K-1)T^2B^2e^{-\lambda} \times \frac{N}{M^2} \sum_{\tau_k=1}^{M} \left[ \frac{\tau_k^2}{\tau_k!} \right] + \frac{N}{M^2} \sum_{\tau_k=M+1}^{2M} \left[ \frac{(\tau_k-M)^2}{\tau_k!} \right] \]

\[ + \frac{1}{\pi} \sum_{\tau_k=1}^{M} \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{1}{(i-j)^2} \left[ 1 - \cos \left( \frac{2\pi (i-j)}{M} \tau_k \right) \right] \left[ \frac{\tau_k^2}{\tau_k!} \right] \]

\[(27)\]

5. Statistical Model of Asynchronous MC-CDMA

With the knowledge of the 1st and 2nd order moments of the total interference, we are able to propose a computer simulation model that can significantly reduce the computational load when simulating asynchronous MC-CDMA systems. The proposed model is shown in Figure 2. It is based on replacing the asynchronous model with a synchronous model followed by an additive noise representing the total interference affecting asynchronous MC-CDMA systems. This allows for a significant reduction of computational load. Indeed, in order to simulate the asynchronous model and evaluate its performance in a given channel, signals from all interfering users with different randomly generated offsets need to be simulated for each reference user. In the proposed model, only reference users are simulated. The requirement for simulation of interfering users is replaced by adding a noise component representing the interference. The noise is zero-mean, white, Gaussian, with a power that has been derived in the previous section.

In next section, the proposed model is validated using Monte-Carlo simulations, by comparing the BER at the output of the asynchronous MC-CDMA system to that at the output of the model. The results are also shown in Table 2.

6. Simulation Results

An asynchronous MC-CDMA system with 8 users using an 8-Length Walsh-Hadamard (WH) Code is simulated. The Rayleigh parameter for the channel is assumed equal to 1. The number of realizations used for the Monte-Carlo simulations is 250000.

6.1 Interference Power

We first validate the results of equations (19) and (27) by computing the interference power in the asynchronous MC-CDMA system using Monte-Carlo simulations for both uniformly distributed and Poisson distributed timing offsets. For Poisson distributed timing offsets, we also computed simulations for different values of the Poisson parameter, i.e. \( \lambda = 0.1 \ T_s \), \( \lambda = 0.5 \ T_s \), and \( \lambda = 0.9 \ T_s \). The results are displayed below.

<table>
<thead>
<tr>
<th>Poisson Parameter</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform Distribution</td>
<td>12.841</td>
</tr>
<tr>
<td>Poisson Distribution</td>
<td>12.8582</td>
</tr>
<tr>
<td></td>
<td>12.4781</td>
</tr>
<tr>
<td></td>
<td>12.9277</td>
</tr>
</tbody>
</table>

It can be seen that equations (19) and (27) provide a very good representation of the interference power in asynchronous MC-CDMA systems for uniformly distributed and Poisson distributed timing offsets respectively.

6.2 Comparison of BER between two models

Using the same simulation conditions described above, the BER performance between the asynchronous MC-CDMA and the proposed computer simulation model is compared. The results are shown in Table 2. It can be seen that for all different distributions of timing offset \( \tau \), the BER error is below 3%.

Although computational time can be different depending on the computer hardware used, we claim that the new model uses approximate one tenth of the
computational time required to compute the conventional asynchronous MC-CDMA system.

### Table 2 – BER performance comparison

<table>
<thead>
<tr>
<th>Normal Distribution</th>
<th>Conventional Model</th>
<th>Proposed Model</th>
<th>Error</th>
<th>Poisson Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poisson</td>
<td>0.1342</td>
<td>0.1303</td>
<td>2.99%</td>
<td>NA</td>
</tr>
<tr>
<td>Poisson Distribution</td>
<td>0.1284</td>
<td>0.1286</td>
<td>0.16%</td>
<td>0.1 T_s</td>
</tr>
<tr>
<td></td>
<td>0.1262</td>
<td>0.1265</td>
<td>0.24%</td>
<td>0.5 T_s</td>
</tr>
<tr>
<td></td>
<td>0.1344</td>
<td>0.1308</td>
<td>2.75%</td>
<td>0.9 T_s</td>
</tr>
</tbody>
</table>

### 7. Conclusion

In this paper, we derived the expression of the conditional power of the MAI and ICI in asynchronous MC-CDMA systems. This expression is given as a function of timing offset $\tau$. Using this result, we derived the 2nd moment of the total interference under two conditions 1) $\tau$ is assumed to be uniformly distributed over $[0, T_s]$; 2) $\tau$ is assumed to be Poisson distributed with its parameter $\lambda$ ranging within $[0, T_s)$. With the knowledge of the 1st and 2nd order moment of the total interference, a computer simulation model is proposed which is proved to have comparable BER performance to the traditional asynchronous MC-CDMA system, but requires much less computational time.

### 8. Reference


