

A COMPUTATIONAL INVESTIGATION OF THE PLASMON MODES OF TESSELLATED ARRAYS

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CERTIFICATE OF ORIGINAL AUTHORSHIP

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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Date: 27-04-2016

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DEDICATION

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Abstract

This study investigates localized surface plasmon resonance (LSPR) phenomena occurring on metallic nanoparticles arranged in tessellated arrays.

Clusters of nanoparticles have interesting optical properties, which could be exploited for enhancement of Surface Enhancement Raman Spectroscopy (SERS) and provide the possibility of tailoring novel technological applications entailing metamaterials for anti-reflecting coatings, superlenses, cloaking and bio-sensing. The geometric arrangement of metallic nanoparticles, and the physical parameters such as size, shape and material, affects the optical response; this leads to significant variation of the transmission spectrum with compared to isolated nanoparticles, and enhanced concentration of energy in the gaps between particles.

Finite element simulations of single and dimer nanoparticles (with an electrodynamic approach) and of a unit cell of tessellated array of different symmetrical shape nanoparticles (with an electrostatic approach) have been performed. The single and dimer results provide a basis for understanding the effect of shape, corner sharpness and gap size on resonant spectra; which provide absorption enhancement and spectral shift according to the geometrical nanoparticle under simulation.

The tessellated arrays are interpreted in terms of the effective permittivity, which is an important material factor that can be continuously tuned, enabling improvement of plasmon-based technological applications. Here we investigate a number of lattices including square and hexagonal arrangements of cylindrical, square, triangular and hexagonal nanoparticles and find that the peaks of resonance and the higher-order modes of the optical response can be mapped thoroughly. Overall these results further develop our perspective of the plasmon modes of arrays and offer some potential for improved tunability of the optical properties of nanocomposite materials. This provides improved strategies for designing meta-materials with specific electric transport and thermal properties.