Analysis of CMC-Supported Embankments Considering Soil Arching

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ABSTRACT: In this paper, the behaviour of geosynthetic-reinforced controlled modulus column-supported embankments is studied for different distributions of loadings induced by arching on the load transfer platform (LTP). This study proposes a mechanical model for idealising the response of LTP-soft soil-column system, by representing each sub-system using commonly used mechanical elements such as rough-elastic membrane, beam, and spring. The soil arching effect is incorporated in the model to determine the deflection of the soft soil as well as mobilised tension in the geosynthetics more accurately. The effects of the column stiffness and consolidation of saturated soft soils are also incorporated in the model. Moment and shear force in the LTP, tension developed in the geosynthetics, and settlements of the improved soft ground are predicted using the proposed model. To evaluate the proposed model, a parametric study is conducted to investigate the influence of different pressure distribution due to different arching theories. It is observed that the pattern of distribution of the arching loads affect the performance of controlled modulus column-supported embankments significantly.

INTRODUCTION

Soft soils underneath the embankments are prone to excessive settlement due to the low stiffness, low bearing capacity as well as high shrink-swell potential. The controlled modulus column (CMC) is one of the ground improvement techniques to meet the higher demand for the transport infrastructure particularly near flood plain comprising weak soil layers. Introducing geosynthetic reinforcement (GR) in column-supported embankments results in a more efficient transfer of load to the columns in the form of an arching mechanism. However, the soil behaviour becomes much more complicated with inclusion of GR. The interaction between the load transfer platform (LTP), the columns, and the soft soil below the LTP change their actual behaviour considerably than what is obtained from the consideration of the each section alone. According to van Eekelen et al. (2013), most of the analytical models for the design of column-supported embankments or LTP with GR include two key calculation steps. In the first step, the arching behaviour in the embankment is calculated and in the second step the load-deflection behaviour of the GR is evaluated. Several analytical models describing Step 1 (arching) are available in the literature. BS8006 (2010) and EBGEO (2010) are two of the limit-state equilibrium models which are frequently used in piled embankment design. van Eekelen et al. (2013) proposed a new model which is an extension of the Hewlett and Randolph (1988) and EBGEO (2010) models. However, it has been observed that very limited studies have been conducted on embankments resting over column-improved soft soil considering mechanistic behaviour of column-supported embankment incorporating the arching effect.

In this paper, a mechanical model has been developed to study the behaviour of CMC-supported geosynthetic-reinforced embankments by considering the effect of soil arching within the embankments, depth of soft soil, stiffness of soft soil and reinforcement layer. The effect of stiffness of the CMC has also been incorporated in the present model. Parametric studies have also been conducted to quantify the effect of different load distribution due to arching on the deflection, bending moment, shear force, and rotation of the LTP as well as the tension developed in the geosynthetic reinforcement.

MECHANICAL MODEL DEVELOPMENT

FIG. 1(a) shows geosynthetic-reinforced embankment resting on CMC-improved soft soil. The model proposed by Yin (2000) for beam on elastic foundation has been modified to simulate the CMC-improved ground by including the effect of CMC. Furthermore, the effects of different soil arching theories within the embankment fill have been incorporated in the analysis. The geosynthetic reinforced granular layer and the soft soil have been simulated with the reinforced Timoshenko beam and the spring-dashpot system, respectively. The CMCs are idealised by stiffer linear springs and the geosynthetics is assumed to be rough elastic membrane as illustrated in FIG. 1(b). The assumed deformed shape of the LTP and the co-ordinate axes for a unit cell are shown in FIG. 2(a). The deformation of the column is assumed to remain unchanged over its width. The CMC and the soft soil are loaded with different distributed loading intensities of p_s and p_c, respectively due to soil arching. In the present study, a 2-D plane strain analysis has been carried out for CMC-supported embankments. Since in the field, discrete columns are placed in a square or triangular pattern, the equivalent plane-strain material stiffness is determined by the relationship suggested by Tan *et al.* (2008) based on matching the column-soil composite stiffness as: $k_{c,pl}a_{r,pl} = k_{c,ax}a_{r,ax} + k_{s,ax}\left(a_{r,pl} - a_{r,ax}\right)$. Subscripts $"pl"$ and $"ax"$ denote plane-strain and axisymmetric conditions, respectively, while a_r is the area replacement ratio. Deformation of the CMC-reinforced composite ground as shown in FIG. 2(a) can be expressed as:

$$w_{xz} = w_{cz} + w_{sz} \qquad \text{for} \qquad 0 \le x \le s'/2 \tag{1}$$

where s' is the clear spacing between the CMCs, w_{sz} is the displacement of the LTP on soft soil region at a horizontal distance x, and w_{cz} is the displacement of the LTP over column region at a horizontal distance ξ.

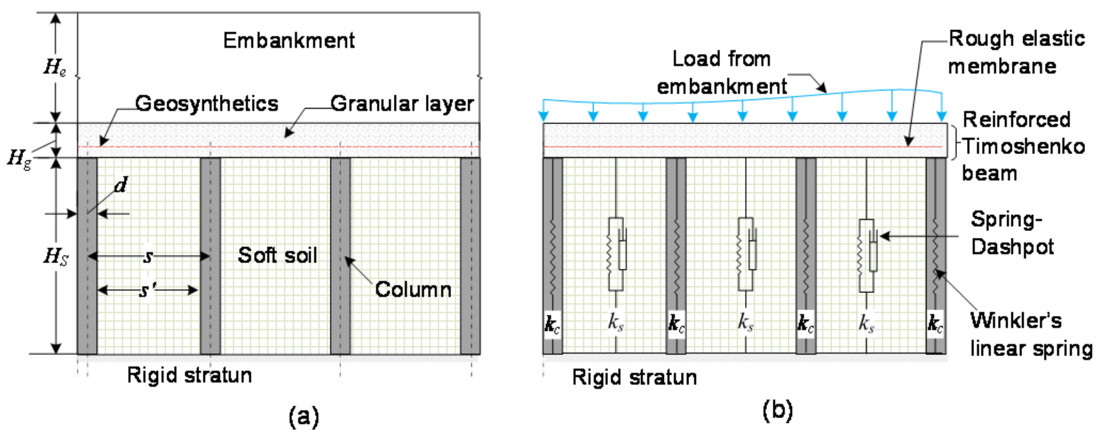

FIG. 1. Sketch of: (a) embankment resting on CMC-improved soft soil and (b) proposed foundation model.

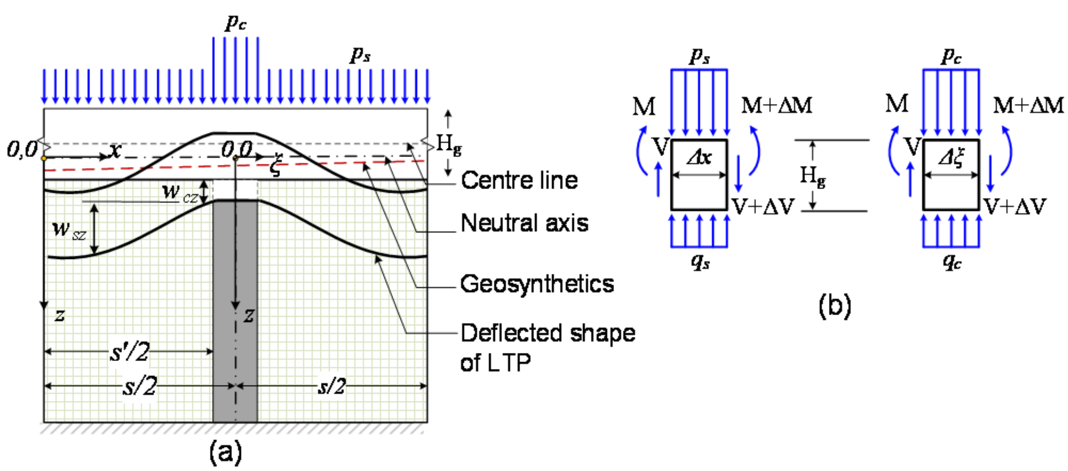

FIG. 2. Illustration of: (a) assumed deformation shape of CMC-improved ground and (b) stresses on the LTP elements within the soil and column region.

To obtain the differential equation, governing the deflected shape of a transversely loaded LTP beam resting on elastic foundation, the LTP is divided into infinitesimal beam elements in the horizontal direction having equal thickness Δx within the soft soil region and $\Delta \xi$ within the column. Typical LTP elements over the soft soil and the columns and the stresses acting on LTP are shown in FIG. 2(b). The equilibrium of the vertical forces and moments of a typical element of LTP for $0 \le x \le s'/2$ under plain strain conditions yield the following equations:

$$\left.\begin{aligned} D_b \frac{d^2\theta_{sz}}{dx^2} + C_b \frac{dw_{sz}}{dx} - C_b \theta_{sz} = 0 \\ C_b \frac{d\theta_{sz}}{dx} - C_b \frac{d^2 w_{sz}}{dx^2} + q_s = p_s \end{aligned}\right\} \tag{2}$$

Similar set of equations can be governed within the CMC region ($0 \le \xi \le d/2$) replacing θ_{sz}, w_{sz}, p_s, and q_s by θ_{cz}, w_{cz}, p_c, and q_c, respectively. In Eq. (2), w is the transverse deformation of the centroid axis of the beam, θ is the rotation angle of the cross section of beam about its neutral axis, D_b and C_b are the bending rigidity and shear rigidity of the LTP with geosynthetics (Yin, 2000). Considering linear stress-displacement relation proposed by Winkler (1867) and consolidation effect of the soft soil suggested by Deb (2007), normal pressure at the LTP-soil (q_s) and LTP-column interfaces (q_c) can be written as:

$$\left.\begin{aligned} q_s = \frac{k_s w_{sz}}{U} \\ q_c = k_{c,pl} w_{cz} \end{aligned}\right\} \tag{3}$$

where U is the degree of consolidation of the CMC-improved soft soil. Although the time dependent behaviour of the soft soil is considered in the proposed model, it should be noted that soil cementation and creep can significantly influence the behaviour of soft soil (Nguyen *et al*. 2014; Azari *et al*. 2014; and Le *et al.* 2015).

Combining Eqs. (2) and (3) and assuming Fourier cosine series which considers the symmetric embankment loading on the LTP, the ordinary fourth-order differential equations of the deflection for the LTP on the soil and column areas can be written as:

$$\left.\begin{aligned} D_b \frac{d^4 w_{sz}}{dx^4} - \frac{D_b k_s}{C_b U}\frac{d^2 w_{sz}}{dx^2} + \left(\frac{k_s}{U}\right) w_{sz} = S_0 + \sum_{m=1}^{m=\infty} S_m \left[1 + \frac{D_b}{C_b}\left(\frac{2m\pi}{s'}\right)^2\right] \cos\left(\frac{2m\pi x}{s'}\right) \\ D_b \frac{d^4 w_{cz}}{d\xi^4} - \frac{D_b k_{c,pl}}{C_b}\frac{d^2 w_{cz}}{d\xi^2} + k_{c,pl} w_{cz} = C_0 + \sum_{m=1}^{m=\infty} C_m \left[1 + \frac{D_b}{C_b}\left(\frac{2m\pi}{d}\right)^2\right] \cos\left(\frac{2m\pi\xi}{d}\right) \end{aligned}\right\} \tag{4}$$

Analytical Solution

The solution of the fourth order nonhomogeneous differential equation governing the deflected shape of a transversely loaded LTP beam resting on foundation soil for $0 \le x \le s'/2$ can be expressed as:

$$\begin{aligned} w_{sz} = e^{\vartheta x}(s_1 \cos\delta x + s_2 \sin\delta x) + e^{-\vartheta x}(s_3 \cos\delta x + s_4 \sin\delta x) \\ + \frac{S_0}{k_s} + \sum_{m=1}^{m=\infty} s_m \cos\left(\frac{2m\pi x}{s'}\right) \end{aligned} \tag{5}$$

Correspondingly, for $0 \le \xi \le d/2$, the governing equation can be expressed as:

$$w_{cz} = d_1 e^{r_1\xi} + d_2 e^{r_2\xi} + d_3 e^{-r_1\xi} + d_4 e^{-r_2\xi} + \frac{C_0}{k_{c,pl}} + \sum_{m=1}^{m=\infty} c_m \cos\left(\frac{2m\pi\xi}{d}\right) \tag{6}$$

where

$$\left\{\begin{aligned} \vartheta = \left(\sqrt{\frac{k_s}{4D_bU}} + \frac{k_s}{4C_bU}\right)^{1/2} \\ \delta = \left(\sqrt{\frac{k_s}{4D_bU}} - \frac{k_s}{4C_bU}\right)^{1/2} \end{aligned}\right., \quad \left\{\begin{aligned} r_1 = \frac{k_{c,pl}}{2C_b} + \sqrt{\left(\frac{k_{c,pl}}{2C_b}\right)^2 - \frac{k_{c,pl}}{D_b}} \\ r_2 = \frac{k_{c,pl}}{2C_b} - \sqrt{\left(\frac{k_{c,pl}}{2C_b}\right)^2 - \frac{k_{c,pl}}{D_b}} \end{aligned}\right., \quad \left\{\begin{aligned} S_0 = \frac{1}{s'}\int_{-s'/2}^{s'/2} p_s\,dx \\ C_0 = \frac{1}{d}\int_{-d/2}^{d/2} p_c\,d\xi \end{aligned}\right. \tag{7}$$

$$\begin{cases} S_m = \frac{1}{s'}\int_{-\frac{s'}{2}}^{\frac{s'}{2}} p_s \cos\left(\frac{2m\pi x}{s'}\right) dx \\ C_m = \frac{1}{d}\int_{-\frac{d}{2}}^{\frac{d}{2}} p_c \cos\left(\frac{2m\pi \xi}{d}\right) d\xi \end{cases} \quad \text{and} \quad \begin{cases} s_m = \dfrac{S_m\left[1+\left(\frac{2m\pi}{s'}\right)^2\frac{D_b}{C_b}\right]}{D_b\left(\frac{2m\pi}{s'}\right)^2\left[\left(\frac{2m\pi}{s'}\right)^2+\frac{k_s}{C_bU}\right]+\frac{k_s}{U}} \\ c_m = \dfrac{C_m\left[1+\left(\frac{2m\pi}{d}\right)^2\frac{D_b}{C_b}\right]}{D_b\left(\frac{2m\pi}{d}\right)^2\left[\left(\frac{2m\pi}{d}\right)^2+\frac{k_{c,pl}}{C_bU}\right]+k_{c,pl}} \end{cases} \tag{8}$$

In practice, the stiffness of the LTP beam is greater than the stiffness of the soft soil and less than the stiffness of the CMC. Hence, the given solutions Eq. (5) and Eq. (6) are valid for $k_s < 4{C_b}^2U/D_b$ (for soft soil region) and $k_c > 4{C_b}^2U/D_b$(for column region). It can be noted that s_1 to s_4 and r_1 and r_2 are the constants of integration which can be determined by applying the boundary conditions. Once deflection (w) of the LTP beam is obtained, rotational angle (θ), shear force (V), bending moment (M) of the LTP, and tension (T) in the geosynthetics can be obtained using the following equations:

$$\left.\begin{array}{l} \theta = \frac{D_b}{C_b}\left(\frac{d^3w}{dx^3} + \frac{1}{C_b}\frac{dp}{dx} - \frac{1}{C_b}\frac{dq}{dx}\right) + \frac{dw}{dx}, \quad M = -D_b\frac{d\theta}{dx}, \\ V = C_b\left(\frac{dw}{dx} - \theta\right), \quad T = -S_{gr}(y_n - y_r)\frac{d\theta}{dx} \end{array}\right\} \tag{9}$$

where S_{gr} is the tensile stiffness of GR, y_n and y_r are the location of neutral axis and GR from the centreline of the LTP, respectively.

Due to symmetry, at the outside boundary of unit cell, the shear stress and the slope are assumed to be zero. It is assumed that there will be no rotation and transverse deformation at the connection between the LTP beam and the column. Within column region, at the column support, shear stress is equal to the reaction from the column support. Since the deformation of the column is constant through the width, the slope will be zero.

Boundary conditions for $0 \le x \le s'/2$ and $0 \le \xi \le d/2$ are as follows:

$$\begin{array}{c} \left\{\left.\frac{dw_s}{dx}\right|_{x=0} = 0, \quad \tau_s|_{x=0} = 0, \quad \theta_s|_{x=\frac{s'}{2}} = 0, \quad w_s|_{x=\frac{s'}{2}} = 0\right\} \\ \left\{\left.\frac{dw_c}{d\xi}\right|_{\xi=0} = 0, \quad \tau_c|_{\xi=\frac{d}{2}} = -p_c + k_{c,pl}w_c|_{\xi=\frac{d}{2}}\right\} \end{array} \tag{10}$$

Following the boundary conditions in Eq. (10), and using Eq. (9) (for θ_s, θ_c, τ_s, and τ_c) constants of integrations can be determined using simple Excel spread sheet. Due to the page limitation, calculation steps are not provided in details.

Geometry and Material Properties of a CMC-Supported Embankments

Parametric study is conducted to predict the vertical deflection of LTP-soft soil-CMC system, bending and shear force in the LTP and tension developed in the geosythetics using the proposed analytical model. This study considers the typical parameters for CMC-supported geosynthetic-reinforced embankments which are presented in Table 1. Modulus of subgrade reaction for the soft soil (k_s) and the CMC (k_c) are estimated using the equations suggested by Selvadurai (1979, Eq. 7.9) for one dimensional settlement.

Table 1. Material properties

Material	Parameters
Embankment fill	H_e = 6m, γ_e = 18.3 kN/m^3, φ_e = 30°
Granular fill	H_g = 0.75m, E_g = 35 MPa, ν_g = 0.3
Foundation soil	H_s = 10m, E_s = 1 MPa, ν_s = 0.3
CMC	H_c = 10m, d = 0.45m, s = 2m (square arrangement) , E_c = 10000 MPa, ν_c = 0.25
Geotextiles	S_{gr} = 1000 kN/m, ν_{gr} = 0.3

Note: H: height/depth/thickness; γ: unit weight; φ: frictional angle; E: Young's modulus; d: diameter; s: spacing; ν: Poisson's ratio; S: tensile stiffness. Suffices e, g, s, c, and gr are used to designate embankment fill, granular fill, soft soil, CMC, and geosynthetic reinforcement (GR).

In this study, it is assumed that the location of GR is always at quarter of the depth of the LTP below the centreline of the LTP. Similar to a concrete beam, a cracked section of reinforced Timoshenko beam (or LTP) is assumed since the soil in the LTP does not carry tension. Therefore, the second moment of inertia and the shear modulus of the beam have been reduced by 25% and 50%, respectively.

Results and Discussion

The stresses acting on top of the LTP layer placed at the base of the embankment can be determined by considering soil arching in the embankment in plane-strain condition using the expressions proposed by BS8006 (2010), EBGEO (2010), and van Eekelen *et al.* (2013). According to BS8006 and EBGEO, stresses are distributed uniformly and in triangular shape on the geosynthetics, respectively. In contrast, van Eekelen *et al.* suggested an inverse triangular distribution of stresses on the geosynthetics. Since, van Eekelen *et al.* provides the minimum load intensity on the LTP within the column as displayed in FIG. 3(a), the minimum downward movements of LTP are observed on the column for this theory as plotted in FIG. 3(b). Whereas BS8006 provides the minimum load intensity on the LTP within the soil region. Hence the minimum downward movements of LTP are observed for BS8006 on the soil area as plotted in FIG. 3(b). Since van Eekelen *et al.* concept results the maximum load at the column-soil interface (Point A in FIG. 4b) shear force generated in the LTP at the column location is the maximum as shown in FIG. 4(a). In contrast, the maximum negative bending moment occurs at the column location in case of triangular distribution of load (EBGEO, 2010) whereas, the uniformly distribution pattern (BS8006, 2010) results in the minimum negative bending moment as plotted in FIG. 4(b). FIG. 5(a) shows the variation of the magnitude of the rotation angle of the cross-section of LTP due to the bending. FIG. 5(b) demonstrates the variation of the geosynthetics tension. It can be seen that the maximum tension occurs at the column edge and then continues to decrease away from the column for all the cases until reaches zero. This is because of the assumption of Timoshenko beam theory that a plane section remains plane after bending. Since, EBGEO (2010) predicts the greatest negative moment at the column edge, the maximum mobilised tension occurs for the case of EBGEO (2010).

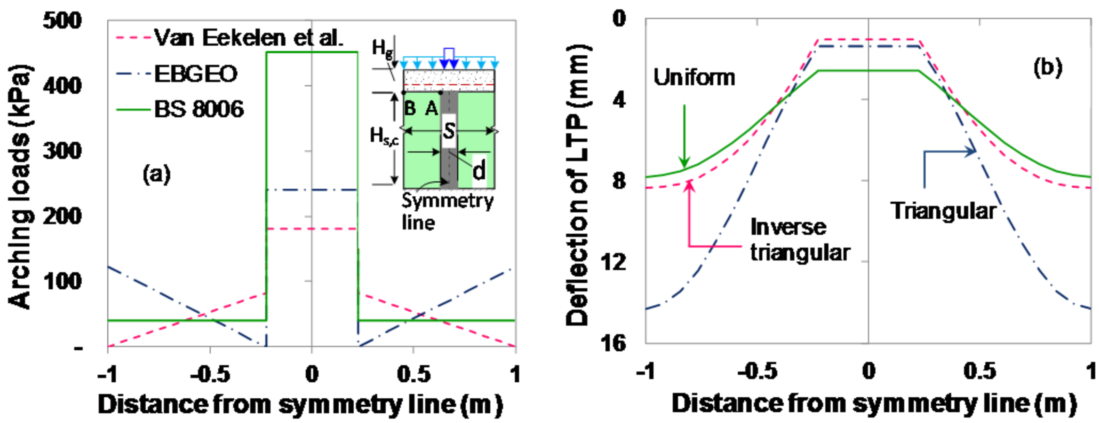

FIG. 3. Effect of arching on: (a) stresses on LTP and (b) deflection of LTP.

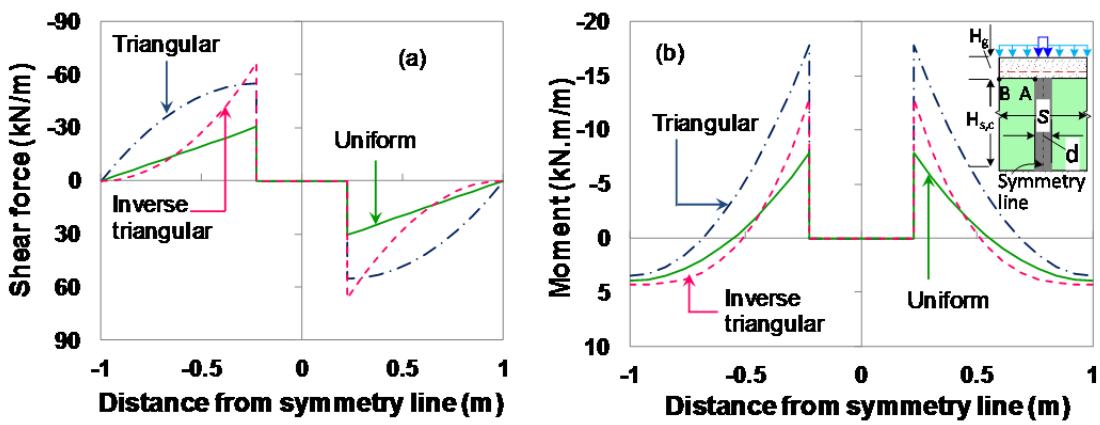

FIG. 4. Effect of arching on: (a) shear force and (b) moment in the LTP.

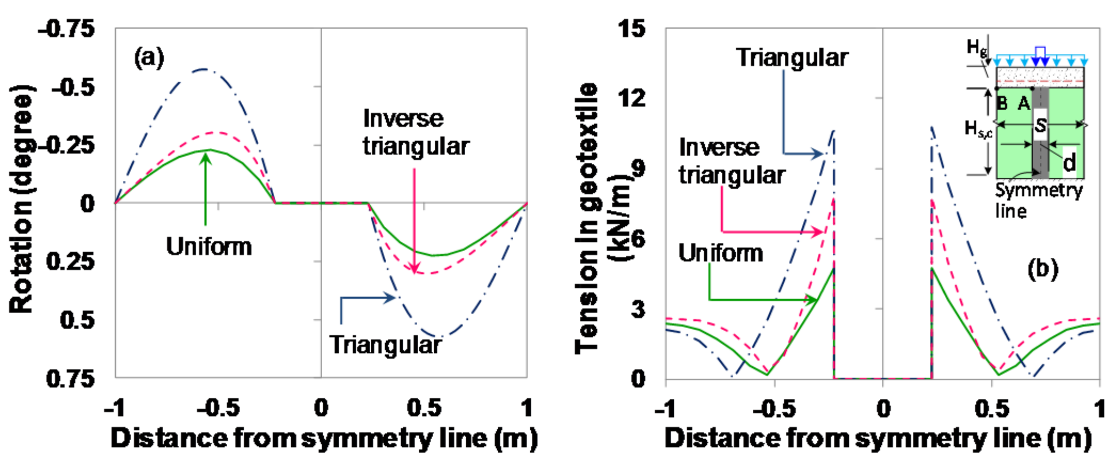

FIG. 5. Effect of arching on: (a) rotation of LTP and (b) tension in GR.

CONCLUSIONS

This study has investigated the effects of different stress distributions patterns on the load transfer platform (LTP) on the behaviour of geosynthetic-reinforced controlled modulus column (CMC) supported embankment simulating the LTP as a "reinforced Timoshenko beam". Results show that the load-settlement response of the LTP-soil-CMC system can significantly be affected by the different arching theories. Uniformly distributed loading on the LTP following the British Standard predicts the smallest settlement, rotation, shear force and negative moment in the LTP within the soft soil region and the smallest mobilised tension as compared with the triangular and inverse triangular distribution of loading. These analysis results certainly raise question mark on the use of British Standard for designing the LTP in particular for the determination of tension capacity of the geosynthetics and settlement of the LTP for CMC improved embankment application.

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